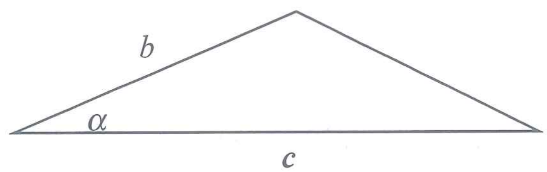
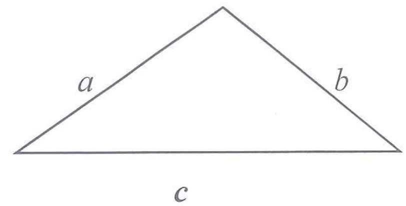
Sec. 7.6 Non Right Triangles—Law of Cosines

Law of Cosines: Can be used to solve oblique triangles when two sides and an included angle are known (SAS) or when three sides are known.



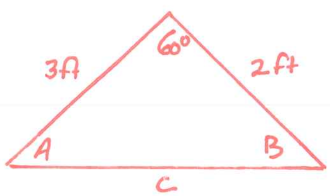


Law of Cosines –

1. $c^2 = a^2 + b^2 - 2ab\cos C$
2. $b^2 = a^2 + c^2 - 2ac\cos B$
3. $a^2 = b^2 + c^2 - 2bc\cos A$

Use Law of Cosines only when you can't use Law of Sine (when you don't have an angle and is opposite side). Use Law of Cosines to find the fourth part of the triangle and then use Law of Sines to find the rest (because it is easier).

Ex: Solve when $a = 2$ ft, $b = 3$ ft, and C = 60°



$c^2 = a^2 + b^2 - 2ab\cos C$
$c^2 = 2^2 + 3^2 - 2(2)(3)\cos 60^\circ$
$c^2 = 4 + 9 - 6$
$c^2 = 7$
$c = \sqrt{7}$
$\boxed{c = 2.646 \text{ ft}}$

$\frac{\sin 60^\circ}{\sqrt{7}} = \frac{\sin A}{2}$

$\frac{\sqrt{7}\sin A}{\sqrt{7}} = \frac{2\sin 60^\circ}{\sqrt{7}}$

$A = \sin^{-1}\left(\frac{2\sin 60^\circ}{\sqrt{7}}\right)$

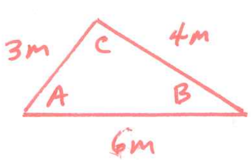
$\boxed{A = 40.893^\circ}$

(has to be smaller than B and must be acute)

$B = 180^\circ - 60^\circ - 40.893^\circ$

$\boxed{B = 79.107^\circ}$

Ex. Solve when $a = 4$ m, $b = 3$m, and $c = 6$m.



$b^2 = a^2 + c^2 - 2ac\cos B$
$3^2 = 4^2 + 6^2 - 2(4)(6)\cos B$
$9 = 52 - 48\cos B$
$-43 = -48\cos B$
$\frac{-43}{-48} = \cos B$
$B = \cos^{-1}\left(\frac{43}{48}\right)$
$\boxed{B = 26.384^\circ}$

Solve for smallest remaining angle as it must be acute!

$\frac{\sin 26.384}{3} = \frac{\sin A}{4}$

$\frac{4\sin 26.384}{3} = \frac{3\sin A}{3}$

$A = \sin^{-1}\left(\frac{4\sin 26.384}{3}\right)$

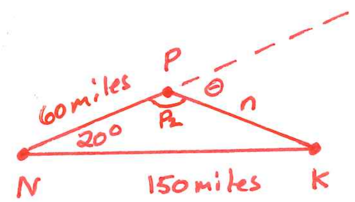
$\boxed{A = 36.336^\circ}$ (must be acute as C is bigger)

$C = 180^\circ - 26.384 - 36.336$

$\boxed{C = 117.280^\circ}$

Ex. A motorized sailboat leaves Naples, Florida bound for Key West, 150 miles away. Maintaining a constant speed of 15 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds, after 4 hours, that the sailboat is off course by 20 degrees. How far is the sailboat from Key West at this time? Through what angle should the sailboat turn to correct its course? How much time has been added to the trip because of this? (Assume that the speed remains at 15 miles per hour.)

$d = rt$
$d = 15(4)$
$d = 60$ miles



$n^2 = k^2 + p^2 - 2kp\cos N$
$n^2 = 60^2 + 150^2 - 2(60)(150)\cos 20°$
$n^2 = 3600 + 22500 - 18000\cos 20°$
$n^2 = 26100 - 18000\cos 20°$
$n^2 = 9185.532826$
$\boxed{n = 95.841 \text{ miles}} \rightarrow$ distance from KW

$$\frac{\sin 20°}{95.841} = \frac{\sin P}{150}$$

$$\frac{95.842 \sin P}{95.841} = \frac{150 \sin 20°}{95.841}$$

$$P = \sin^{-1}\left(\frac{150 \sin 20°}{95.841}\right)$$

$P_1 = 32.364$
or
$P_2 = 180 - 32.364$
$\rightarrow P_2 = 147.636°$ Must be obtuse angle based on scenario. We normally solve for smaller remaining angle (K) to eliminate need to choose as it must be acute.

$$\frac{\sin 20°}{95.841} = \frac{\sin K}{60}$$

$$\frac{95.841 \sin K}{95.841} = \frac{60 \sin 20°}{95.841}$$

$$K = \sin^{-1}\left(\frac{60 \sin 20°}{95.841}\right)$$

$K = 12.364°$
$P = 180° - 20° - 32.364°$
$P = 147.636°$

$\theta = 180 - P_2$
$\theta = 180 - 147.636°$
$\boxed{\theta = 32.364°}$

$d = rt$
$150 = 15t$
$10h = t$

$d = rt$
$60 + 95.841 = 15t$
$155.841 = 15t$
$10.389h = t$

$\boxed{.389 \text{ hours or } 23.364 \text{ min}}$

HW: pg 305-308, #2,10,28,31,37,~~45~~